Fitting the term-structure in STAMP 8.10

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1 Introducing STAMP 8.10

In the Summer of 2008 we have upgraded STAMP to version 8.10 which is the current release version. The new items in version 8.10 are relatively small. The forecasting dialog allows the forecasting of the unobserved components as well as the future observations. More importantly, various errors have been removed from the program and some improvements have been introduced. Most notably, the batch facilities have improved and some bugs in the routines for models with explanatory variables and intervention variables have been solved. Finally, the automatic outlier and break detection procedure has been optimized further.

for the professional use of state space methods in econometric, statistical and general time series analysis. The state space form provides an unified representation of a wide range of linear Gaussian time series models including autoregressive moving average (ARMA) models, time-varying regression models, dynamic linear models and unobserved components time series models. State space methods are used in many different fields including forecasting, signal extraction, seasonal adjustment, business cycle analysis, macroeconomic models, option pricing, financial analysis based volatility, interest rate term-structures (yield curve) and many more.

To emphasize the powerful options available in STAMP 8.10, we would like to present a multivariate analysis of the interest rate term structure in this OxMetrics Newsletter.

2 Term structure

Fitting and predicting time series of a cross-section of yields has proven to be a challenging task. For many decades work on the term structure of interest rates has mainly been theoretical in nature. In the early years work focused on the class of affine term structure models, see Vasicek (1977) and Cox et al. (1985). It has been shown that the forecasts obtained from this class of models do not outperform the basic random walk forecasts, see for example Duffee (2002). These findings may not be very surprising given their focus on the cross-section dimension of yields without a reference to the time series dimension. Time series models aim to describe the dynamical properties and are therefore more suited for forecasting.

The papers of Diebold and Li (2006), DL, and Diebold et al. (2006), DRA, have recently shifted attention to the time series dimension by generalizing the Nelson and Siegel (1987) model. DL and DRA introduce the dynamic Nelson-Siegel model as a statistical three factor model to describe the yield curve over time. The three factors represent level, slope and curvature of the yield curve and thus carry some level of economical interpretation. More importantly, DL and DRA show that the model-based forecasts outperform many other models including standard time series models such as vector autoregressive models and dynamic error-correction models. In DRA, the Nelson-Siegel framework is extended to include non-latent factors such as inflation. Further they frame the Nelson-Siegel model as a multivariate unobserved components time series model with three common factors which are modeled by vector autoregressive processes. A wide range of statistical methods associated with the state space model can be exploited for maximum likelihood estimation and signal extraction, see Durbin and Koopman (2001). We will follow their approach to some extent and show that `STAMP` 8.10 can be used for a yield-curve analysis of different interest rates associated with different maturities.

3 The `STAMP` model for the yield curve

Interest rates are denoted by $y_t(\tau)$ at time t and maturity τ. For a given time t, the yield curve $\theta_t(\tau)$ is some smooth function representing the interest rates (yields) as a function of maturity τ. A parsimonious functional description of the yield curve is proposed by Nelson and Siegel (1987) and is based on three common factors with pre-set weights for each maturity such that the factors can be interpreted as the level, slope and shape of the yield curve. In our `STAMP` analysis, we also adopt three unobserved common levels (factors).

In case we observe a series of interest rates $y_t(\tau_i)$ for a set of N different maturities $\tau_1 < \ldots < \tau_N$ available at a given time t, `STAMP` can estimate the yield curve for the multivariate model

$$y_t(\tau_i) = \mu_i + \lambda_{1i}\beta_{1t} + \lambda_{2i}\beta_{2t} + \lambda_{3i}\beta_{3t} + \varepsilon_{it}, \qquad (1)$$

for $i = 1, \ldots, N$, where μ_i is a constant, β_{jt} is the jth factor (at time t) and λ_{ji} is the weight for the jth factor associated with the ith maturity. The disturbances

$\varepsilon_{1t}, \ldots, \varepsilon_{Nt}$ are assumed to be independently distributed with mean zero and constant variance σ_i^2. In the Nelson-Siegel framework, the weights are restricted. In STAMP we estimate the weights λ_{ji} by maximum likelihood.

The three factors in the vector $\beta_t = (\beta_{1t}, \beta_{2t}, \beta_{3t})'$ can be modeled by the vector autoregressive (VAR) process as in DL and DRA. In empirical work on the yield curve, the factors of the term structure are often found to be very persistent. Therefore, we consider a random walk process for β_t as given by

$$\beta_{t+1} = \beta_t + \eta_t, \qquad \eta_t \sim NID(0, \Sigma_\eta), \tag{2}$$

for $t = 1, \ldots, n$, with variance matrix Σ_η and initial condition $\beta_1 \sim N(0, \kappa I)$ where $\kappa \to \infty$. Further details of the model are discussed in the STAMP manual.

4 Data

For our STAMP analysis of yield curves, we consider the unsmoothed Fama-Bliss zero-coupon yields dataset, obtained from the CRSP unsmoothed Fama and Bliss (1987) forward rates. We analyze monthly U.S. Treasury yields with maturities of 3m, 6m, 9m, 12m, 15m, 18m, 21m, 24m, 30m, 3y, 4y, 5y, 6y, 7y, 8y, 9y and 10y (m=months, y=years) over the period from January 1972 to December 2000. This dataset is the same as the one analyzed by DL and DRA. We notice that for our dataset, we require model (1) and (2) with $n = 348$ and $N = 17$. This illustration shows that STAMP can handle such a high-dimensional model and is able to estimate the parameters by maximum likelihood.

5 Results from STAMP

We follow section 6.4 of the STAMP manual since our yield curve model (1) and (2) is similar to the model in equation (6.3) of the manual. STAMP is used first to estimate the coefficients λ_{ji} for $j = 1, 2, 3$ and $i = 1, \ldots, N$ (51 parameters), the variances σ_i^2 (17 parameters) and variance matrix Σ_η (6 parameters). For identification purposes, the weights are restricted as implied by equation (6.4) in the manual. In our case, the weights of the three factors and for the maturities 3m, 24m and 10y are restricted to be $1, 0, 0$ and $0, 1, 0$ and $0, 0, 1$, respectively. The 3m yield is therefore exclusively related to the first factor while the 24m and 10y yields are exclusively associated with the second and third factors, respectively. We estimate the parameters (in total 65) by the method of maximum likelihood. The constants μ_i and the factors β_{it} are estimated via the Kalman filter smoothing algorithms which are computational efficiently implemented in STAMP.

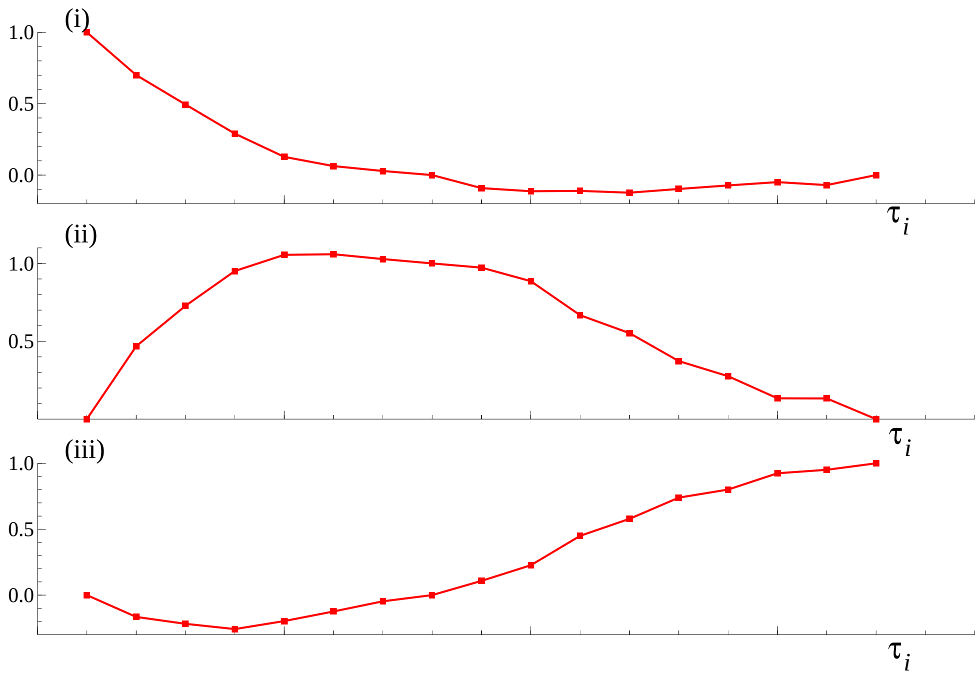

Figure 1: The estimated weights for the three factors associated with the 17 maturities.

Although much output is presented in `STAMP`, here we can only present a selection. The estimated weights associated with the factors β_{1t}, β_{2t} and β_{3t} are displayed in Figure 1. The maximum likelihood estimates of the weights for a particular factor are presented against maturity and these appear quite smooth. We therefore observe that the weights (or loadings) for the first factor are associated mostly with the short term (3m – 15m) maturities. In the same way, we conclude that the second factor is mostly associated with the medium term (1y – 4y) maturities and the third factor represents the interest rates for (5y – 10y). The estimate of the variance matrix of the disturbances η_t in (2) is given by

$$\widehat{\Sigma}_\eta = \begin{bmatrix} & \text{3m} & \text{24m} & \text{10y} \\ \text{3m} & 0.3720 & 0.8378 & 0.6265 \\ \text{24m} & 0.2638 & 0.2664 & 0.8685 \\ \text{10y} & 0.1306 & 0.1533 & 0.1169 \end{bmatrix},$$

where the upper triangular of the matrix contain the implied correlations. The three factors are all correlated with each other. The correlations between the short-term and medium-term factors and the medium-term and long-term factors are higher compared to the correlation the correlation between the short-term and long-term factors.

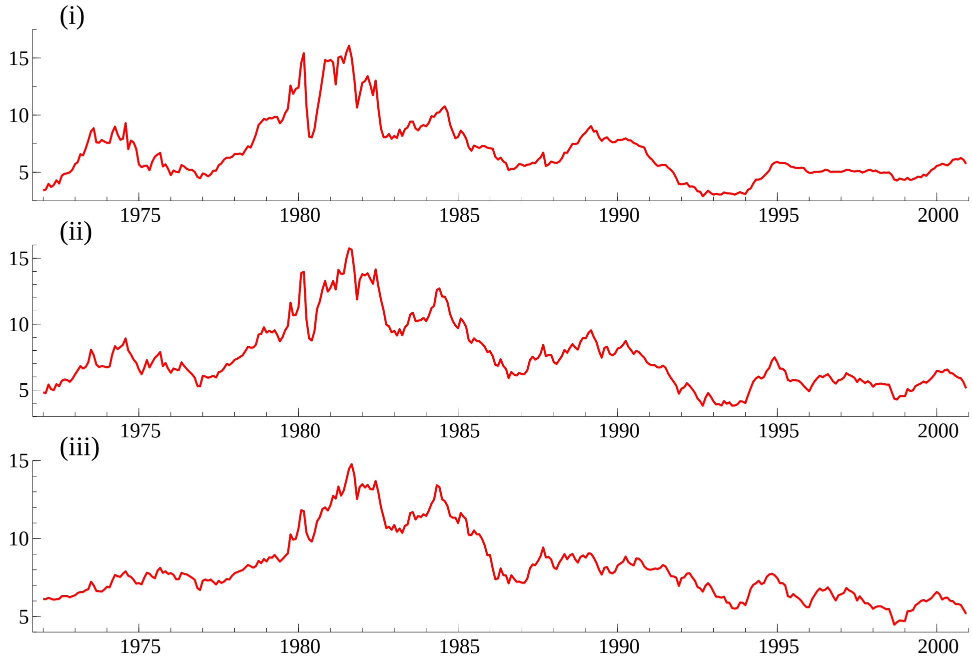

Figure 2: The three estimated factors: (i) $\hat{\beta}_{1t}$, (ii) $\hat{\beta}_{2t}$, (iii) $\hat{\beta}_{3t}$

The smoothed estimates of the three factors (obtained from the Kalman filter and smoothing algotithm implemented in `STAMP`) are presented in Figure 2. The three estimated factors look similar although there are local differences. By observing the three estimated factors over time, the variability of the short-term factor is higher than the long-term variability.

Finally, `STAMP` is, and has always been, used to forecast time series (as part of modeling and analysis). In case of our illustration, we present the one-step ahead predictions of the 17 yields in 2000 in Figure 3. It is encouraging that this basic model for the yields, provides an accurate set of predictions for each month.

References

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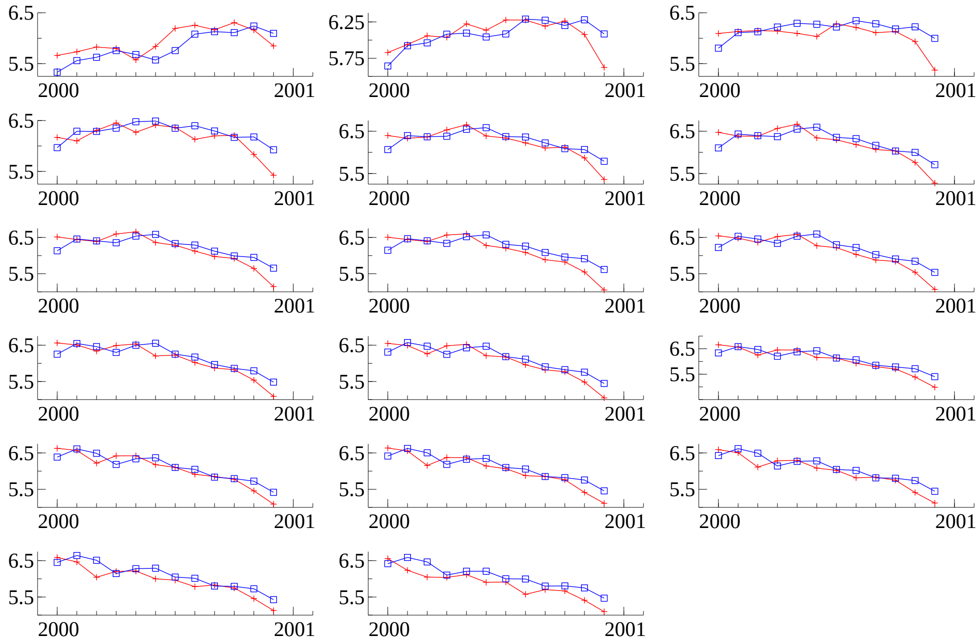

Figure 3: One-step ahead forecasts of US forward rates in 2000 for the 17 maturities.

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